DOCUMENT RESUME

ED 210 843 EC 140 643

AUTHOR Fleischner, Jeannette; Garnett, Katherine
TITLE Arithmetic Learning Disabilities: A Literature Review. Research Review Series 1979-80. Volume 4.
INSTITUTION Columbia Univ., New York, N.Y. Research Inst. for the Study of Learning Disabilities.
SPONS AGENCY Bureau of Education for the Handicapped (DHEW/OE), Washington, D.C.
PUB DATE 80
CONTRACT 300-77-0492
NOTE 83p.

EDRS PRICE MF01/PC04 Plus Postage.
DESCRIPTORS *Arithmetic; Elementary Education; Generalization; *Learning Disabilities; Literature Reviews; *Mathematics Materials; Minimal Brain Dysfunction; *Remedial Mathematics; *Teaching Methods; Textbook Evaluation

ABSTRACT

The authors review literature on conventional and special approaches to arithmetic instruction with emphasis on applying these approaches to the teaching of elementary learning disabled students with disorders in the area of arithmetic. Section I focuses on the instructional context of arithmetic. It is suggested that generalization teaching (which makes full use of "binding principles," or reasoning schemes applicable across groups of combinations) might be most profitable if the thinking strategies for teaching combinations were made more explicit for disabled students. Section II reviews four traditional arithmetic texts (Growth in Mathematics, Field Mathematics Program, Holt School Mathematics, and Silver Burdett Mathematics) in terms of how these different series treat the development of proficiency in the basic number combinations of addition, subtraction, and multiplication. A third section on special approaches addresses structural arithmetic, arithmetic for the brain injured child, procedures proposed by Cruickshank, the Fernald arithmetic remediation method, individually prescribed instruction, and Distar Arithmetic. Section IV reviews characteristics of 10 programs which 36 educators proposed as useful for teaching arithmetic to children with minimal brain dysfunction. A final section discusses developmental dyscalculia, factors in arithmetic learning disabilities, and basic number facts. A bibliography with approximately 300 references completes the document. (SB)

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ARITHMETIC LEARNING DISABILITIES:

A LITERATURE REVIEW

Jeannette Fleischner
Katherine Garnett

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Research Review Series 1979-1980
Volume 4

Series Editor, Elizabeth Neuman

Funded under OE/BEH Contract Number 300-77-0492

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INTRODUCTION

The literature on mathematics mastery which is relevant to understanding the problems of learning disabled children is clouded by a number of basic issues which have not achieved resolution. While a treatise on the psychology of mathematical learning is clearly beyond the intent of this review, several of these issues are crucial to understanding the complexities of mathematics mastery for learning disabled children.

When lay persons, or teachers, speak of children's difficulties in mathematics, they most often refer to problems associated with arithmetic, which is viewed by mathematicians and psychologists alike as a special case of mathematics (Brainerd, 1973; Farnham-Diggory, 1972; Cohn, 1971; Chalfant and Scheffiin, 1969; Gagne and Paradise, 1961). As Johnson and Rising (1972) put it, "Mathematics is a creation of the human mind, concerned primarily with ideas, processes, and reasoning . . . mathematics is a way of thinking, a way of organizing a logical proof. It can be used to determine whether or not an idea is true, or whether it is probably true" (p. 3).

In addition, mathematics is a language (Johnson and Rising, 1972). It is a system of symbolic representation which uses ideograms to represent concepts and ideas. The ideograms of mathematics, numerals, represent a universal language among members of complex societies, according to Cohn (1971). The syntax of mathematics is represented by the set of signs which designate the rules governing combinations, which permit manipulation of ideas. The semantic content of mathematics is, or can be, as simple or as complex as the ideas which are being represented. Thus, 3+2=5 represents in mathematical terms a relatively

simple notion, whereas $E=mc^2$ represents a notion so complex that few achieve full understanding of it.

Arithmetic has been viewed by Chalfant and Schefflin (1969) as a ". . . branch of mathematics that deals with real numbers and their computation (p. 119). . ." According to Buffie, Welch and Paige, (1968) the objectives of the mathematics curriculum are:

1. Development of computational facility.
2. Development of mathematical understanding.
3. Development of rational power (including divergent, logical, and creative thinking).
4. Development of a positive attitude toward mathematics.

It is of interest to the learning disabilities specialist to note the pre-eminent position accorded computational facility, for failure to develop this facility is the first indication that a child is experiencing difficulty in mastering arithmetic. In fact, it has been argued that computational facility in addition, subtraction, multiplication and division is a necessary, but not sufficient, condition for total mathematical competence. (National Council of Teachers of Mathematics Yearbook, 1978; Thornton, 1977; Brownell, 1935.)

Basic texts in learning disabilities mourn the paucity of literature on the nature and treatment of mathematics disabilities (Bryan and Bryan, 1978; Hammill and Bartel, 1978; Lerner, 1971). In an effort to draw upon literature relevant to this area, one is guided by the admonition of Chalfant and Schefflin (1969) that disabilities in <u>mathematics</u> may be distinct from disabilities in <u>arithmetic</u>--that the cognitive processes required for mathematical reasoning may only partially intersect the processes required for success in arithmetic.

Yet, when teachers of elementary school aged children are questioned, and when teachers of learning disabled children are questioned, consensus is quickly achieved that it is proficiency in arithmetic that is a worrisome problem among learning disabled children.

Brainerd (1973) puts the failure to master the mathematics curriculum into a social context:

> The significance to society of number and number-related skills has increased tremendously with the rise of industrial civilization . . . During the first few years of elementary school, roughly 50% of the curriculum is normally given over to inculcating the natural numbers and methods of manipulating them. The child learns to add, subtract, multiply and divide the natural numbers, and finally to derive other number systems from them . . . We expect our children to possess by pubescence a numerical competence much higher than that of an educated Greek or Roman adult of two millenia ago. We have even gone so far as to develop labels that imply mental turpitude on the part of those otherwise normal children who fail to attain the standards of achievement we deem desirable, for example, "learning disability" and "underachiever" (p. 104).

This statement underscores the cries of learning disabilities specialists who cite the lack of evidence dealing with the nature of and correlates of arithmetic learning disability. In fact, one cannot but be struck, on reviewing the literature, by the paucity of empirically derived evidence regarding mathematical learning disability. In particular, it is striking when contrasted to reading, which has attracted a cadre of persons interested in the processes which underly both ability and disability in reading (Levin and Williams, 1970; Smith, 1976).

The problems involved in understanding disability in mathematics are further complicated by the absence of a well-documented sequence for normal learners. Piaget (1950, 1952(a and b), 1953, 1957, 1963, 1970, 1973) is, of course, well known for his numerous postulations of the invariant stage-sequence through

which children pass in order to attain mature cognitive powers. Frequently he cites mathematical reasoning as an example of the cognitive development of which he speaks. He rarely, however, speaks directly about arithmetic as the special case of mathematics (Piaget, 1970, 1973). It remains for others to deduce the curriculum and instructional implications from his theories.

Sister Gilmary, I.H.M. (1970) summarizes Piaget's conclusions about mathematics learning:

1. That the child learns independently and spontaneously; that when the learning is imposed it becomes entirely verbal.
2. That the child must grasp the concept of conservation of quantity before he can grasp the conservation of number and, hence, the concept of number.
3. That since number notion is a logical notion, the child needs to be able to discover relationships as a prerequisite to the formation of any concept of number.
4. That in the development of the idea of conservation, the child goes through the stages: (a) no conservation; (b) a phenomenistic, unstable sort of notion of the conservation based on the observation and manipulation of different forms and transformations; and, finally, (c) a logical, axiomatic certainty of conservation in the case of all transformation.
5. That the child attains these respective stages at the approximate ages of: (a) 4½-5; (b) 5-6; (c) 6-7½.
6. That although the child can "learn to count," that is, say the number names, on a one-to-one basis for objects in a series, as early as five, he/she is unable to do even this if the perceptual framework is altered.

7. That mathematical concepts are built up from using concrete materials, but are independent of the actual materials used.
8. That grouping and the additive concept are developed by the child at approximately the same time as the concept of number (p. 285).

Investigators have confirmed, in many cases, the stage-sequence postulated by Piaget (Brainerd, 1973; Churchill, 1958; Dodwell, 1960, 1961; Elkind, 1961; Feigenbaum, 1963; Lovell, 1961; Lunzer, 1956; Wohlwill, 1960; Smedslund, 1964). It is fair to say that Piagetian theory has guided the research on the development of mathematical concepts during the past twenty years. The research which is derivative of Piaget's own has focused largely on the general stages of cognitive development associated with particular mathematical concepts, rather than on particular skill acquisition. Thus, it has been demonstrated that a child ". . . who is a nonconserver is likely to experience great difficulty in understanding the concept of addition" (Wheatley, 1970, p. 294). Hood (1962) found that high/low rankings on a test of arithmetic abilities was directly related to the status of a child as a conserver or nonconserver. This finding was confirmed by Williams (1958). Other aspects of mathematical readiness within a Piagetian framework have been enumerated by Bartel in Hammill and Bartel (1978). They include the ability to classify, to seriate, and to place objects in one-to-one correspondence. In addition, understanding of space and spatial representation has been identified as crucial to arithmetic mastery. Perhaps central to arithmetic operations is flexibility and reversibility of thought . . . the child must understand that when an operation has been performed, it can be reversed. When it has been reversed, the original quantity will have been conserved.

There is little or no evidence on which to base an assumption that learning disabled children are, as a group, deficient in the conceptual bases of arithmetic mastery, as those bases are suggested by Piagetian theory. Should such evidence emerge, it would be necessary to pursue arithmetic instructional methods which incorporated many of the activities presently associated with pre-school curricula rather than elementary school curricula.

ARITHMETIC: THE INSTRUCTIONAL CONTEXT

Despite the profound change of emphasis evident in the proliferation of modern mathematics curricula, a basic pedagogical dilemma persists: the dilemma of "finding the most adequate methods for bridging the transition between these natural but nonreflective structures to conscious reflection upon such structures and to a theoretical formulation of them" (Piaget, 1970, p. 47). This perspective is reiterated numerous times throughout Piaget's discussion of the relations among psychology, mathematics and teaching methods. It would strongly suggest that, though much has been made of the 'meaning' aspects of arithmetic education since the classical work of Brownell (1928, 1935a, 1935b), modern mathematics, at least in its practiced form, has far from resolved its basic pedagogical dilemma of how to go about this charge. There remain serious debates regarding appropriate teaching procedures for promoting in children a meaning-filled and functional grasp of number relations and operations.

Modern mathematics has been lauded as representing progress in principle, but has also found serious criticism in many of its practical expressions (Skemp, 1971; Kline, 1973; Piaget, 1970). While the various strands which make up the teachings of modern mathematics purport to further a child's understanding, they have in practical application often confused that which they were attempting to

teach and have in many cases provided insufficient practice at each stage of teaching. The problem would appear to center on the balance between instruction in a meaningful context and mastery of the component skills and fundamentals of arithmetic.

Basic facts or number combinations have assumed an important position in arithmetic teaching because proficiency at this early level of computation has been consistently perceived to be fundamental to further arithmetic study (Ashlock and Washbon, 1978; Brownell, 1935). In a recent review of research concerned with computation, mastery of basic facts was first in a listing of the pivotal skills necessary for computational competence within each whole number operation (Suydam and Dessart, 1976). Additionally, basic number facts are deemed important because "they provide simple, 'small number' contexts for the development of mathematical ideas pertaining to an operation" (Suydam and Weaver, 1975, p. 57). A recent study of sixth graders (Baxter, 1974) reports that errors of greatest frequency were found to be in basic facts and in regrouping. Numerous other studies have undertaken analyses of the computational errors that children made (Ellis, 1972; Lankford, 1972; Smith, 1969, Flourney, 1967, Smith and Eaton, 1949; Grossnickle and Snyder, 1939; Buswell and John, 1926). Results of these studies employing written tests, oral interviews, diagnosis of student's work and tape-recorded explications show evidence of seven major error categories. Three of these categories refer to errors within the computing of basic facts: 1) basic fact errors for each operation, 2) errors involving zero for each operation and 3) errors of subtracting the minuend from the subtrahend. Concern for children's effective knowledge of basic arithmetic facts would appear to be well founded.

The basic facts within each of the four whole number operations are considered to be all those number relations which result in sums from 0 to 18,

differences from 0 to 18, products from 0 to 81, and quotients from 0 to 9. They include 100 separate facts for each of the four arithmetic operations. Acquisition of proficiency with the addition and subtraction basic facts is one of the fundamental goals of arithmetic teaching in grades one and two. Mastery of basic facts in multiplication and division is considered one of the major tasks of instruction for grades three, four and five. Despite the varying approaches which have been taken to elementary level arithmetic, there appears to have been consistent unanimity regarding the central significance of an adequate mastery of these elementary number facts.

> Since memorization of basic facts is an absolute necessity for proficiency in mathematics, it is important to stress reinforcement until memorization is achieved. After adequate preparation with sets and manipulative devices, frequent drills which challenge the students' memories should be an integral part of the learning process (Duncan, Capps, Dolciani, and Zweng, 1967, p. 113).

It is of interest to note this emphasis given to drill work within the framework of a 'modern math' set-theory approach. This emphasis can be viewed as the legacy of one of the four basic influences discernable in the various arithmetic methodologies currently in practice: Drill Theory, Incidental Learning Theory, Meaning Theory and Generalization Theory.

During the first half of the century, this country saw the rise of Drill Theory, propelled by Thorndike's (1898) notions regarding associative bonding. The one hundred basic facts within each of the four whole number operations were parsed into small, discrete and unrelated units. Teachers were admonished to drill each combination to proficiency, avoiding contamination of one combination by another, related one (Morton, 1927). This discreteness was seen as paramount, since otherwise each bond would be weakened and would not be available to being

called on automatically and without confusion. Arguments favoring this independence of each basic fact were many (Roantree and Taylor, 1932; Studebaker, Knight, and Findley, 1929) and even included strong statements denying the existence of interrelationships between facts (Knight and Behrens, 1928).

In sharp contrast to this drill-connectionist orientation which dominated arithmetic teaching well into the 1930's, two alternative theoretical perspectives grew in formulation and found their way into the mainstream of teaching practice. The first, known as Incidental Learning Theory, was concerned with what was considered to be the three major aspects of the learning situation: A child's readiness for particular arithmetic learnings, the primacy of a child's motivation, and the social usefulness of what was being taught (Washburne, 1930; Wilson, 1926). Children were delayed in their introduction to number operations and the study of arithmetic was included only incidentally as an outgrowth of particular activities or projects. In the early grades a specific fact or arithmetic process was to be taught only as the need for it arose. Though residual effects continue to be felt, school programs operating within the Incidental Learning Theory framework received serious criticism for their disregard of the necessarily sequential nature of arithmetic learning (Brownell, 1935). While critics such as Brownell and Hanna (1935) generally eschewed incidental learning practice, they praised its awakening of concern for social utility and the child's need for purpose.

William Brownell, considered the father of modern mathematics teaching (Suydam and Weaver, 1970), was the first to elaborate what he termed Meaning Theory. Brownell's theory conceived of arithmetic "as a closely knit system of understandable ideas, principles, and processes." His early work focused

on the development of children's number ideas in the primary grades (Brownell, 1935, 1928). He observed a correspondence between the ease with which a dot configuration was apprehended and the number of dots exposed. This correspondence faded as children became older and came to rely on more mature forms of number grouping. Interviewing students regarding the various methods they used, Brownell discerned a developmental sequence consisting of four stages. He described the first level as the counting stage. The second stage was seen to be partial counting, wherein the child takes one group as a whole, adding the remaining units to this base group. The third level was distinguished by subgrouping as when, for example, given the configuration ⁘ ⁖ the child subgrouped 6+2+2 in order to arrive at an answer. Thinking in multiples characterized the fourth stage: given ⁖ ∴ the child conceived of two 4's, less one. Brownell pointed out that in most cases the method a child employed was consistent, irrespective of the presentation form of the number work. This meant that children who used partial counting to ascertain the number of dots in a configuration, also used partial counting when presented with numerals. The children in Brownell's study were being taught number facts by stringent drill methods, but, in actuality, were not learning in the associative manner expected. Instead, they were discovering and employing systems which made use of meanings and relationships. Apparently, coming to "know" a number fact involves a period of development, a discovery of the relationships inherent in quantitative thinking, an organization of meanings.

In summarizing their research with third graders, Brownell and Chazal (1935) concluded that drill must be preceded by instruction which is meaningful. The early work of Brownell and Carper (1943), more recently confirmed by the studies of Jerman (1970), found that in computing multiplication combinations

children make use of different strategies for different combinations. Additionally, it would appear that certain combinations have heuristics associated with them which are presumed to facilitate their retention. These combinations which appear to develop a heuristic base are n x 0, n x n, n x 5, n x (n-1), and n x (n+1). The facilitation effect is observed in the comparatively lower response latencies for combinations of these types (Jerman, 1970). Another recent study (Wood, Resnick and Groen, 1975) makes use of response latencies in addressing the question of how second and fourth grade children process simple subtraction problems. The results of this study point toward a developmental trend, the characteristics of which suggest that children progress from an algorithmic model to more heuristic procedures. This would imply that some considerable time after initial learning takes place, children normally develop more efficient means of processing basic subtraction facts and that these means involve some sorts of strategies which differ from the computing methods that were originally taught to them.

A further extension of the basic notions found in Meaning Theory are evidenced in the work of Thiele. In his explication of Generalization Theory, Thiele (1938) notes that there is considerable agreement between his conceptions and those of Brownell. He describes the major difference as having to do with the much greater importance given to larger generalizations in his own framework. In Meaning Theory the frequent verification of the sum for a single combination is classified as the perception of a generalization. Except for the commonality of ten in higher decade problems, Meaning Theory makes little use of the larger generalizations. Though the ensuing studies and programs of Wheat (1937), McConnel (1934), and Olander (1931) include an appreciation for

more of these generalizations, they do not organize number learning experiences around them. Generalization Theory, in contrast, makes full use of "binding principles," (Dewey, 1910, p. 82) or reasoning schemes, which apply across groups of combinations.

The basic tenets of Meaning Theory have clearly exerted the most dominant influence on current teaching practice in elementary level arithmetic. As has been noted, the contribution of Drill Theory continues to find a place in present day programs, though its assertion that repetitive practice alone results in the optimum and most efficient learning has long been discarded (at least one hopes that this is actually so in practice). The praiseworthy strengths of Incidental Learning Theory have found their way, in various forms, into the different strands of modern mathematics, while this theory's disregard for arithmetic's inherently systematic nature has been abandoned. It can be said that, starting with Meaning Theory, a major emphasis in all modern teaching of basic arithmetic facts has been on the development of understanding. Many unresolved issues concerning how to best facilitate the development of understanding are expressed in programs with greatly divergent orientations: counting approaches (Nuffield Project, 1967); sets approaches (Patrick Suppes, 1965); and structural approaches (Catherine Stern, 1949 and Cuisinaire, 1972).

The Task Force undertook a survey of major arithmetic series currently in common use. It suggests that Generalization Theory has exerted only a minimal effect. The commutative property ($5 + 3 = 3 + 5$), the distributive property ($4 \times (5 + 7) = (4 \times 5) + (4 \times 7)$), and the common law of adding ten are the only generalizations which receive consistent attention in these series. The order, or sequence, employed in presenting number facts to children is based upon early research concerning the relative difficulty of the different number facts.

Investigations into the relative difficulty of basic number combinations received considerable focus during the time that Drill Theory was the most prominent underpinning of teaching practice. A review of the difficulty-ranking studies of Clapp (1934), Knight and Behrens (1928), Washburne and Vogel (1928), and Batson and Combellick (1925), reveals only partial agreement regarding several general factors which affect level of difficulty: 1) zero combinations are usually found to be at a high level of difficulty, 2) combinations and their reverses are not generally more difficult, and 3) larger addends increase the level of difficulty. Results from these and other ranking studies have been used by textbook authors and classroom practitioners to determine the order in which facts will be presented. The logic behind sequencing the teaching of facts according to their level of difficulty is that, thereby, memorization can be facilitated. Arguing from the results of their own investigations, Swenson (1944, 1949) and Thiele (1938) assert that relative difficulty can be shown to be, at least in part, a function of method of teaching. The Task Force's survey of current arithmetic texts revealed that little focus is given to the relations among number facts. The basic notion, put forth and tested by Thiele (1938) and Swenson (1949), regarding the benefits of grouping facts to be taught according to the generalizations which apply to them has not guided major arithmetic curriculum writers of our day.

Very recent research concerned with basic number combinations (Rathmell, 1978; Thornton, 1978) has returned to a focus on those generalizations, or relationship strategies, upon which Swenson and Thiele founded their work. Many of the strategies employed by the children in Rathmell's investigation paralleled those used by students in this earlier research (for example,

counting on, making use of doubles, thinking one more or less than a known combination, employing the commutative property, using ten). Rathmell (1978) reports a strong relationship between high achievement on addition fact tests and use of the more mature strategies for finding addition sums. Superiority of retention of facts over time is also reported for those children using more mature number-relationship strategies. Interviews with regular and special class children (Myers and Thornton, 1977) support Rathmell's findings, while further noting that, unlike their regular class counterparts, the learning disabled youngsters tend not to discover for themselves or to make consistent use of strategy relationships when these are not specifically taught. Ashlock (1971) has stressed that teaching and activity time needs to be spent on developing these thinking strategies as a crucial middle step towards achieving mastery.

As a further exploration of the effectiveness of resequencing the instruction of basic facts, Thornton (1978) compared strategy teaching to traditional number-fact teaching procedures, using regular-class second and fourth graders. The younger children worked on addition, followed by subtraction facts, while the fourth graders studied multiplication and then division basic facts. The results on post-test and retention testing markedly favored the strategy groups for all four operations, despite the fact that the pre-test addition and subtraction means for the strategy-taught second graders were significantly lower than those for the traditionally-taught group. An analysis of interviews conducted with a sample of children from this study reveals that strategy-taught children, both the second-graders and the fourth-graders, were, in fact, making use of what they had been taught. In addition, strategy-taught children appeared

to have made considerably greater use of a wider range of generalizations than did the traditionally-taught youngsters.

The resurgent interest in generalization-teaching, evidenced by these recent investigations, has confirmed earlier work, demonstrating once again that normal learners benefit significantly from basic fact teaching which is organized around specific thinking strategies. Rathmell (1978), Myers and Thornton (1977), Ashlock (1971), and others have suggested that children who exhibit difficulties with elementary arithmetic might also benefit from generalization-based instruction. These authors further suggested that such instruction might be most profitable if the thinking strategies for teaching combinations were made more explicit for disabled youngsters.

ARITHMETIC INSTRUCTION: CONVENTIONAL APPROACHES

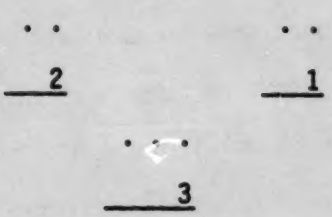
Four of the most widely used traditional arithmetic texts were reviewed (Growth in Mathematics, 1978; Field Mathematics Program, 1974; Holt School Mathematics, 1974; Silver Burdett Mathematics, 1973). This review focuses on how these different series treat the development of proficiency in the basic number combinations of addition, subtraction and multiplication. The term basic number combinations, or basic facts, refers to addition problems up to and including 9 + 9, subtraction problems up to and including 18 - 9, and multiplication problems up to and including 9 x 9. For a typical program scope and sequence see Hammill and Bartel, 1978.

The teacher's editions accompanying each of the texts describes the overall method of presentation as 'task-analytic.' Each series attempts to break down the tasks into their component subtasks, which are then sequenced from smaller to larger for ease in mastering each task. Each lesson is one or two pages in length, with only one or two essential subtasks. Concise objectives are stated for each page and suggested supplementary activities are included, though clearly, these are used only at the discretion of the individual teacher. Review pages are provided at the end of the sequence for each task and these are suggested as useful for diagnostic and remedial work. The amount of skill testing is, in general, left up to the teacher.

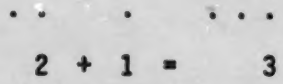
The organization of each page is characterized by lines drawn for separation, often with illustrative examples set off in boxes. An introductory section usually contains one illustrative example. After the mastery of a skill is presumed, problems are placed in rows or columns.

Addition and Subtraction

Each of the programs begins its section on addition by joining two pictures sets. The children are then requested to tell how many members are thereby represented:



This introductory work is covered in approximately two pages, six examples to a page. The subsequent two pages include the addition sign and the equal sign:



This is followed by two pages in which the sum illustration is removed. The final phase deletes all the illustration and has the children draw their own representations. Such illustrative sequences appear throughout the texts at the start of each new task, but the number of examples becomes considerably fewer.

The first chapters on addition are contained within approximately fifteen pages, with a six to seven page focus on the joining of sets and a gradual transition to basic facts. All the facts covered have sum of five or less. The zero rule, or identity element, is introduced at this time, but strictly within the context of these small addend combinations. The identity element is most often shown by a sequence of combinations illustrated by dot configurations:

. . .	. .	3 + 2 = 5
. . .	.	3 + 1 = 4
. . .	0	3 + 0 = 3

A dot pattern is also included showing the left addend changing. The illustrations are then followed by a review page containing approximately twenty-five unillustrated basic fact problems with sums of five or less. At this juncture some of the books make their first presentation of the commutative property, illustrating this within several pages containing from six to ten paired examples:

. . .		. .
3	+	2 = 5
. .		. . .
2	+	3 = 5

The other texts consistently present paired problems such as these throughout their entire addition section, starting with the union of sets, through to the end of the chapter. All of the texts do, by one means or another, highlight the teaching of the commutative property of addition.

The final section of each addition chapter consists of unillustrated problems, presented first horizontally and then vertically. One or two pages of chapter review, or chapter testing, are also included. The number of problems ranges from twenty to thirty and covers sums of five and less, the commutative property, and vertically presented examples.

The placement of beginning subtraction within the teaching sequence and

the manner of introducing subtraction are consistent across the various textbook series reviewed here. Immediately followirg the review pages for sums of five and less, subtraction is introduced with an illustration sequence paralleling that for addition. The illustrations visually represent members of a group leaving or being crossed out. Initially, the student simply inserts numerals beneath each picture, then, as with addition, the subtraction sign and the equal sign are included. Gradually the accompanying illustrations are phased out. The inclusion of the identity element for subtraction completes the subtraction sequence. This sequence only covers subtraction problems with factors from zero through five.

Following a review lesson containing from twenty to thirty subtraction problems, approximately four pages of related addition and subtraction problems are presented:

$$3 + 1 = 4 \qquad\qquad 4 - 1 = 3$$

The mixed-problem pages progress from illustrated pairs of related addition and subtraction to different pairs which are unillustrated and finally to a mix of these two groups of paired problems. After a chapter test reviewing all the addition and subtraction facts in both vertical and horizontal form, several of the texts shift their teaching focus to topics other than basic facts (i.e., time, measurement, place value). Within the substantial number of pages devoted to these other topics, there are spaced reviews of basic facts every ten to fifteen pages.

The second major grouping of basic facts includes addition problems whose sums range from six to ten. The sequence for introducing these does not parallel

the sequence employed with the smaller addend combinations. Whereas the zero to five combinations were presented separately and more or less randomly, the six to ten facts are introduced sequentially according to their sum. This approach has been termed a number-family approach. Two pages of illustrated addition problems each of whose sum is six are followed by one page of unillustrated problems resulting in the same sum. These pages combined usually cover the commutative pairs for all the members of the six family. There then follows a review page consisting of approximately fifteen unillustrated combinations with sums from zero to six. Thus, after work with that family of addends whose sum is six, these facts are practiced with those smaller addend facts previously taught. This same pattern is continued for the family of seven, eight, nine and ten.

Usually included along with this chapter are the subtraction facts whose minuends range from six to ten. They are presented in the same manner as the addition facts from six to ten, with approximately the same number of practice problems. The texts vary as to whether they present all the mid-range addition facts before going on to the mid-range subtraction combinations or whether they intersperse addition families of six with subtraction families of six, proceeding accordingly with the families of seven, eight, nine and ten. In either case, after sums from zero to ten and subtraction examples with minuends ranging from zero to ten have been completed, there follow several pages of review using unillustrated problems along with workpage activities such as matching columns of problems to the appropriate answers. These review practices then lead to a chapter test containing twenty-five to thirty problems.

At this point, the texts once again shift their teaching focus to other

arithmetic topics (time, measurement, money, counting by two's), including basic facts in spaced reviews every ten to fifteen pages. Preparatory to work with the basic facts from eleven to eighteen, each text spends a substantial number of pages on the concept of place value. The 'harder' basic facts (with sums and minuends from eleven to eighteen) are introduced and practiced in the same manner as were the earlier mid-range facts (sums and minuends from six to ten). The only variation occurs in the pacing: the harder facts are introduced at a progressively faster rate. The final pages of this section review the basic addition and subtraction facts within two pages, each containing twenty-five to thirty problems. This is followed by a similarly structured chapter test.

Multiplication

In their presentation of basic multiplication facts, the texts begin in one of two ways: the array method or the repeated addition method. Both are utilized by all the books: their order of presentation differs.

Several pages of arrays of various objects are presented. Underneath the columns, the student is to put the number of objects in that column and add those totals. Beneath this line is another, illustrating the grouping idea of multiplication as repeated addition.

. .
. .
. .

3 + 3 = 6

2 threes = 6

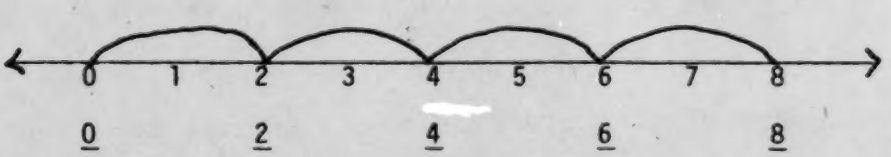
There are approximately two pages of this type of exercise, containing ten questions per page. The next task maintains the illustrations but, at this point introduces the multiplication sign:

. . .
. . .
2 + 2 + 2 = 6

3 x 2 = 6

Here there are approximately two pages of this type of exercise, consisting of ten problems per page.

At this point the emphasis is shifted to a number line concept. (In some books the number line concept is presented before the arrays.) The student has to skip-count (or jump) by two's:



4 jumps of 2 is 8
4 x 2 = 8

This extended number line goes up to 18 and there are generally several pages of horizontal multiplication problems (two is always a factor). The student may use the number line as a reference in these problems. This concept is extended to counting by fives with some forty problems using the new number line.

The books generally return to the array method at this point to reinforce the concept of multiplying by two and five. The students are then shown how to make a table to remember the facts.

2	4	6	8	10	12	14	16	18

They complete approximately two pages of multiplication problems (both vertically and horizontally) totalling about thirty problems.

The same method (number of pages and examples) is used to extend knowledge of the multiplication facts to three and four. After completing the tables for two, three, four and five, the student is given two pages of facts (horizontally and vertically) to review. At this point the commutative property of multiplication is generally illustrated in a set of exercises showing related pairs:

$$3 \times 4 = \underline{12}$$

$$4 \times 3 = \underline{12}$$

Approximately ten to fifteen pairs of problems are presented in this way for the purpose of reinforcing the concept. Illustrations are generally used at the beginning of this exercise to represent the relationship. The next factors introduced are zero and one, generally presented through the array method. There are fewer problems following this section (only one page of twenty problems).

Most of the texts end a chapter at this point and have several pages of multiplication review (twenty to thirty problems per page) before a chapter test consisting of thirty problems with arrays, number lines and basic multiplication facts from zero to five.

The following chapter begins with a large multiplication table (9 x 9) and illustrates for the student how few facts are still left to learn.

Chart of Multiplication Tables

1	2	3	4	5	6	7	8	9
2	4	6	8	10	12	14	16	18
3	6	9	12	15	18	21	24	27
4	8	12	16	20	24	28	32	36
5	10	15	20	25	30	35	40	45
6	12	18	24	30				
7	14	21	28	35				
8	16	24	32	40				
9	18	27	36	45				

Left to learn

There is a group of multiplication problems after the presentation of the table to reinforce how it is to be used (approximately 20-25 problems). The rest of the chapter consists of the presentation of the six, seven, eight and nine facts in the same method previously described for zero to five. There is virtually no difference in the number of exercises for each section or the frequency of review pages. After the presentation of multiplication by nine, there are two or three pages of approximately thirty multiplication facts and these are followed by a chapter test of the basic multiplication facts (both horizontally and vertically). Overall, the multiplication facts are covered in approximately fifty to sixty pages.

ARITHMETIC INSTRUCTION: SPECIAL APPROACHES

Despite the far-reaching effects of arithmetic disability, relatively little attention has been focused on problems within this academic area. There are comparatively few programs, tests, materials or methods for assisting children who exhibit arithmetic learning disabilities (Hammill and Bartel, 1978).

It appears clearly evident that the resourcefulness of indivudal teachers plays a pivotal role in a learning disabled child's progress in mathematical understanding proficiency and application. Within this area of relative paucity of research and development activities, there emerge two general instructional thrusts: the first consists of highly structured program sequences, such as Distar (1972) and IPI (1976); the second consists of specially created techniques and methodologies, such as those proposed by Cruickshank (1961) and Strauss and Lehtinen (1947). The Structural Arithmetic Program of Catherine Stern (1971, 1949) would seem to blend the major aspects of both these instructional thrusts, though it has received criticism on several points. In particular, its over-reliance on only one form of concrete representation, its less-than-adequate provisions for practice and the high cost involved in purchasing its materials for groups of children have been the center of criticism and appear to be the major reasons for its not having been more widely adopted.

Structural Arithmetic

The structural arithmetic program of Catherine and Margaret Stern was developed to address the early arithmetic needs of young primary-grade children. Though it has not been widely used by school systems in general in this country, (Sawyer, 1971), it has become one of several programs commonly used with learning disabled children (Padula, 1975). The Stern method rests on specific guided experiences with a highly structured set of concrete devices. Through systematic experiences with these materials, children are expected to discover visuo-spatial patterns and to "gain insight into arithmetic" (Stern and Stern, 1971, p. 18). The foundation materials for the program are a series of segmented rods, each of

a different color and graduated length. These rods provide a concrete physical model, analagous to the series of natural numbers. Counting skills receive minimal attention, as the system is based on notions of visual Gestalt formation and is meant to be taught as a measurement approach. For the initial teaching of addition facts, combinations are grouped by the families which result in sums from one to ten. The mastery sequence of instruction for each operation is similar to that used by Thornton (1978) and Thiele (1938). Drill activities are expressly forbidden, as they are considered a serious obstruction to the development of mathematical thinking. Teachers are repeatedly assured that after many insightful activities with structural materials, children will find each number element or sequence of number elements 'unforgetable.' Repetition of visuo-spatial patterns is presumed to form indelible pictures in the child's mind. Review of the literature reveals no empirical evidence regarding the efficacy of this program, either for young normally-developing children or for learning disabled youngsters.

Arithmetic for the Brain-Injured Child

The guides and methods proposed by Strauss and Lehtinen (1947) were also developed in accordance with the psychological outlook of Gestalt psychology. From this viewpoint, the aim of arithmetic instruction is the development in the child of a scheme of visuo-spatial organization with emphasis on the interrelationships of the various parts of the scheme. Strauss and Lehtinen consider that the brain-injured child lacks the ability of the normal child to discover spontaneously those significant relationships of the number system. Though disturbance in visual perception is not viewed as the sole component of arithmetic difficulty for these children, it is beleived to be the most seriously

difficult one to overcome.

Strauss and Lehtinen specify four general principles which underlie their proposed instructional procedures:

1. Number concept, based originally upon organized perceptual experiences, depends upon the relationships of objects in space and the resulting development of a number scheme.

2. Development of such a scheme is the outgrowth of ability to organize; it is a semi-abstract structure evolved from the understanding of relationships, of parts to parts and parts to the whole.

3. This visuo-spatial scheme will be abstracted from its perceptual concrete origins when relationships are grasped and meanings understood. This implies the need for perceptual experiences (concrete or semi-concrete) until such organization occurs.

4. For the child with organic disturbances, it may often be necessary to develop special material and techniques of instruction based on our knowledge of such disturbances (Strauss and Lehtinen, 1947, p. 152).

Specially developed materials are suggested to reduce perseverative tendencies and distractability and to focus the child's attention to one aspect at a time. Examples of these are a counting box with slots for large wooden pegs, a screw board, a slide rule counter with cut-out holes, an abacus, and special number boxes. It is stressed that, while these specially-designed materials and techniques are crucial to the development of a basic understanding of number relationships, it is necessary to employ them in progressively more sophisticated ways so that children develop more and more mature means of handling number situations. The development of these more mature ways of thinking about numbers is considered to be a necessary prerequisite for the introduction of the more complex number operations.

Cruickshank

The arithmetic sequence and procedures proposed by Cruickshank (et. al. 1961) and his associates were developed for use with brain-injured and hyperactive children. Underlying their developmentally sequenced program of instruction is

the notion that form perception is basic to the ability to view the world in quantitative fashion. Six guidelines are offered, highlighting the particular features of this program:

1. The child will need many manipulative materials to help overcome motor disinhibition and perseveration.
2. The worksheets must call for only the amount of work the child is able to do successfully.
3. Each new factor must be repeated over a longer period of time, and it must be varied in as many ways as possible.
4. The page of a commercial workbook is so crowded and full that the child cannot tolerate it.
5. The purposeful use of color on each teacher-prepared paper will help the child attend to the task before him.
6. He must be given a carefully calculated starting point and ending point to help him establish himself in space (Cruickshank et. al., 1961, p. 207).

Since color plays a significant role in this program's methodology, color activities constitute the beginning stage of the child's number work. Color-coded dot configurations form the basic, teacher-made, physical representation which the children use as they progress through the sequence. Other suggested materials include colored dowels of different lengths with rods for standing them upright and in relation to one another, a color-coded abacus, and the colored rods from the Stern Arithmetic Program (1971). The instructional sequence follows closely those of common texts currently in use with normal children, but includes at each step the supplementary aids of color and a stress on pictorial configurations. It is considered that as a part of the sequence, many activities, materials and worksheets will need to be constructed by the classroom teacher as each unit of study is initiated.

Fernald Arithmetic Remediation Method

Though the methods described by Grace Fernald (1943) do not, in a strict sense, constitute a system of arithmetic teaching, they do exemplify many of the arithmetic teaching practices commonly employed by teachers of learning disabled children. Two assumptions run implicitly throughout the descriptions of particular techniques: 1) that which is not known by a child must be isolated from that which is known, so that the range of targets for remediation is delimited and, 2) varieties of concrete physical representations are necessary to enable children to derive answers in a way that makes meaningful the substance of what they are doing. The variety of materials suggested include beans, money, rulers, stones, shells and sticks (for bundling in making tens). Emphasis is placed on the pretesting of children in order to determine the sources of difficulty. To a large degree, it is presumed that the incorrectly answered example is, in and of itself, the difficulty which the child faces. An adequate amount of experience with the elements of a particularly problematic example is considered to be the major mode of remediation. Those combinations which are problematic are isolated on separate sheets of paper. If, in the course of verifying combinations by using concrete materials, children make use of counting short-cuts, these are encouraged. But, teachers are strongly admonished not to make suggestions regarding various thinking short-cuts. After various experiences with physical representations of quantity, substantial written practice ensues. The charting of each child's individual progress is a key feature of Fernald's methodology. Initial charting encourages students by displaying to them what they already know as distinguished from what it is they need to work on. The continuation of progress charting is viewed as useful as record-keeping for the teacher and as a success indicator for the child. The

copying of troublesome problems after verification is another integral part of the instruction, though this copying is meant to remain meaningful and is therefore accomplished in small increments over the time of study. The use of money is highlighted as a useful vehicle for learning number combinations. No empirical investigations have been undertaken regarding the efficacy of these proposed remedial methods.

Individually Prescribed Instruction (IPI) - Mathematics

The Individually Prescribed Instruction Program holds as its central tenet that individualizing both instructional goals and resources will provide for individual differences. Instructional resources are keyed to the many highly specific instructional objectives. Students are allowed to pace themselves and are encouraged to become self-directed as quickly as possible. The program is comprised of forty units, each of which focuses on a cluster of related objectives. There are several levels within each of the forty units, each corresponding roughly to grade levels. The teacher selects the unit within which the student will work, and then administers that unit test which determines the entry level of the child. This initial program testing determines specifically which objectives have already been met by the student and which comprise his prescribed section of work. Upon completion of all the specified objectives at that level a mastery test is administered and the student either proceeds along the continuum or is "recycled" through that portion of the program. This is a highly comprehensive instructional program covering the range of basic elementary school arithmetic skills.

Distar Arithmetic I, II, III

The approach taken in the Distar Arithmetic Kits embodies the philosophy that highly systematic and intensive structuring of those skills necessary for

eventual mastery within basic subject matter areas is the most efficient means of teaching. Each of the Distar Arithmetic Kits contains a Teacher's Guide, a number of Teacher's Presentation Books, individual workbooks for each child and, according to the level, additional materials such as form boards, fact cards and card stands. The highly structured teaching procedure requires the teacher to present to the child precisely what is to be said or done. The teacher's actions and words are detailed by the program and are highly specific. Pupils are required to respond orally (written responses are required more extensively in the revised kits) and the pacing of exchanges is expected to maintain a rapid fire level in order to promote continuing interest and attention. The skills and sub-skills are sequenced from simple to more complex and the objectives of the program and mastery criteria are clear and explicit. Common mistakes are predicted in the teacher's guide and specific responses for correcting these errors are provided.

CHARACTERISTICS OF REGULAR PROGRAMS WHICH MAY BE ADAPTED FOR SPECIAL USE

A study (Padula, 1975) designed to identify and critically examine instructional programs used in teaching arithmetic to children with minimal brain dysfunction, reveals 10 programs which thirty-six educators propose are useful for such children. The investigator designed and applied an evaluative instrument to each of these ten programs. Significant characteristics of each program were identified and assessed.

The ten programs identified by the special educators were:

1. Stern, C., Stern, M. B., and Gould, T. Structural Arithmetic. New York: Houghton-Mifflin Company, 1965. (Developed to teach elementary school arithmetic through use of highly structured manipulative materials.)

2. Montessori, M. Montessori Developmental Mathematics Sequences. Holland: Association Montessori International, 1918. (Developed to teach elementary school arithmetic through use of intersensory approach with specifically created equipment.)

3. Cuisinaire, G. Opening Doors in Mathematics. New York: Cuisenaire Company of America, 1972. (Developed to teach elementary school arithmetic through use of specific structural materials.)

4. Dienes, Z. P. Dienes Modern Mathematics Experience Program. Toronto: Herder and Herder Company, 1969. (Developed to teach elementary school arithmetic in a flexible manner, providing teachers with guides and suggestions for real-life problem-solving activities.)

5. Englemann, S. and Carnine, D. Distar. New York: Science Research Associates, 1972. (Developed to teach elementary arithmetic by use of verbal presentation.)

6. Gunlack, B., et al. Understanding Numbers. New York: Laidlow Brothers, 1968. (Developed to teach elementary school arithmetic through a variety of traditional approaches.)

7. Lennes, N. J. and Traver, L. R. Essentials of Arithmetic. New York: Laidlow Brothers, 1964. (Developed to teach elementary arithmetic through extensive drill activities.)

8. Eicholz, R. and O'Daffer, P. Elementary School Mathematics. Palo Alto, California: Addison-Wesley, 1973. (Developed to teach elementary school arithmetic through a variety of traditional approaches.)

9. Hancock, J. and O'Brien, J. A. S. M. D. Palo Alto, California: Addison-Wesley, 1963. (Developed to teach addition, subtraction, multiplication and division of whole numbers through the use of programmed materials.)

10. Matthews, G. Nuffield Foundation Mathematics. New York: John Wiley and Sons, 1972. (Developed to teach elementary arithmetic through use of laboratory approach with emphasis on the discovery process.)

From the results of applying the evaluative instrument to each of these ten programs, the author concludes there are at present no programs geared specifically to the needs of learning disabled children. The characteristics of the programs are described so that comparisons can be made among programs and adaptations can be made by individual teachers.

Tests of Arithmetic Ability and Disability

Two categories of published tests are useful in assessing the arithmetic ability of learning disabled children. The first, standardized achievement tests, permits evaluation of the relative standing of the student in comparison to age or grade mates. The second, diagnostic arithmetic tests, permits analysis of the deficits which may underlie poor achievement in arithmetic. In addition, many authors have suggested that analysis of errors made on tests of computational skill can reveal the sources of confusion, and are therefore useful as an adjunct to standard testing.

Standard Achievement Tests

Standard batteries for the assessment of school achievement routinely contain subtests on arithmetic attainment (Thorndike and Hagen, 1978). It

is common to find that achievement batteries assess arithmetic achievement with regard to arithmetic concepts, problem solving ability, and computational skill. The arithmetic achievement scores for each of these subtests is then averaged to create a composite score. In addition, the students' relative ability in each of the component areas is assessed by differential performance on the subtests. For instance, a student might be quite competent at computation, but do poorly on arithmetic application, or problem solving. Without additional information about that student, it is impossible to determine the cause of poor performance on problem solving; deficits in the ability to think analytically might contribute to poor performance, but so might inability to read the word problems presented. This argument is presented simply to reinforce the notion that standardized achievement batteries serve only a gross screening function in evaluating arithmetic learning disabilities. Poor performance on a standard achievement test signals that further investigation is necessary in order to determine the nature of the student's problems.

Table 2 presents a listing of commonly used standardized achievement batteries which include arithmetic subtests. Review of these tests leads to the conclusion that most are of limited use for learning disabled children, for no opportunity exists to evaluate the process which a child employs in arriving at an answer. This results from the format of the tests, which depend on the selection of the correct response from an array of several alternatives.

There are several diagnostic mathematics tests (see Table 3) which are designed to provide teachers and specialists with more specific information on the skills which students have mastered and those which have yet to be mastered. Three of these commonly used diagnostic instruments are The Key Math Diagnostic Arithmetic Test (Connolly, Nachtman, and Pritchett, 1971),

The Diagnostic Chart for Fundamental Processes in Arithmetic (Buswell and John, 1925), and The Stanford Diagnostic Mathematics Test (Beatty, Madden, Gardner, and Karlsen, 1976).

The Key Math Test is an individually administered diagnostic test designed for use with children in kindergarten through eighth grade. Its authors state that there are no upper limits for use with remedial students. The instrument makes use of an easel format, with answer choices provided. It is organized into three distinct areas (content, operations and applications) and includes fourteen subtests:

Area I - CONTENT

1. Numeration
2. Fractions
3. Geometry and Symbols

Area II - OPERATIONS

4. Addition
5. Subtraction
6. Multiplication
7. Division
8. Mental Computation
9. Numerical Reasoning

Area III - APPLICATIONS

10. Word problems
11. Missing elements
12. Money
13. Measurement
14. Time

The Key Math Test is a comprehensive arithmetic battery which provides an overall indication of a child's arithmetic skill, while at the same time detailing more specific information for teaching specific skills. It yields four levels of diagnostic information: overall performance, area performance, subtest performance, and by-item performance. While differences in area score performance can provide some guiding information regarding relative strengths and weaknesses in content, operations and application of knowledge, the by-subtest pattern analysis suggested by Connolly et al. has been called into question (Salvia and Ysseldyke, 1978). Salvia and Ysseldyke propose that the by-item interpretations provide the most useful level of information, since the specific behaviors sampled by each test item are keyed to specified behavioral objectives for teaching. Making use of a student's score at this fourth level of interpretation is considered to provide extremely valuable diagnostic information, while avoiding the real shortcomings inherent in basing inter-individual and intra-individual comparisons on the available normative data.

The Buswell and John Diagnostic Chart does not provide final scores, quotients, or grade equivalents. It is an individually administered measure which provides specific information regarding the methods a child uses in solving arithmetic examples. Observation of a child's work is the method of diagnosis. The Diagnostic Chart, the Pupil's Worksheet, and the examples described in the Manual of Directions comprise the components of the instrument. During administration, the child uses the Pupil Worksheet which consists of a graded series of written arithmetic problems, arranged in order of increasing difficulty in each of the four fundamental processes (addition, subtraction, multiplication and division). The child is directed to work each problem aloud, so that the examiner can ascertain what methods are

being employed and wherein the thinking process deviations leading to incorrect answers may lie. The Teacher's Diagnostic Chart provides spaces for noting the child's problem-solving habits. The teacher refers to the list of habits in interpreting the methods used for each example and then summarizes the errors which are made consistently over several problems. The Manual of Directions then provides suggestions for remediating poor arithmetic work habits. Though the Diagnostic Chart was published over 50 years ago, it remains a useful instrument for present-day diagnostic purposes (Wallace and Larsen, 1978). Its diagnostic usefulness is solely as a criterion-referenced measure within the circumscribed domain of written whole-number operations.

The Stanford Diagnostic Test is a group administered measure comprised of four levels: Red (grades 1.5-4.5), Green (grades 3.5-6.5), Brown (grades 5.5-8.5), and Blue (grades 7.5-high school). Each of the four levels contains three subtests: Number System and Numeration, Computation, and Applications. The Stanford yields norm-referenced scores such as grade equivalents, percentile ranks and stanines. The "Progress Indicators," '+' or '-' indications of whether a child has reached or exceeded a predetermined cutoff score within a particular area, are not set in accordance with strict norm-referenced procedure and are thereby considered to be the criterion-reference aspect of the instrument (Salvia and Ysseldyke, 1978). Given the excellent standardization procedures employed, strengths and weaknesses can be reliably pinpointed among the specified areas of skill development (Salvia and Ysseldyke, 1978). Administration of this test takes several group sittings and error-type analysis is severely limited because the answer format is multiple choice.

TABLES

TABLE 1

Hierarchy of Cognitive Strategies Underlying

Arithmetic Basic Fact Computation

I. Counting Strategies

Count all Units

Partial Counting

Count up from larger unit in addition

Count down in subtraction

II. Linking or Solution Strategies

Link response to already known fact

(Ex: 6 + 7 = 6 + 6 + 1)

Solve problem by partitioning

(Ex: 8 + 6 = 8 + 2 + 4; 8 + 6 = 6 + 6 + 2)

III. Habituation

Response is automatized

No intermediary steps are used when responding to problem, although earlier strategies are available for reconstruction of answer should retrieval of response be delayed.

TABLE 2

Achievement Tests With Arithmetic Subtests

Name of Test	Grade Level	Skills Measured
California Achievement Tests (Tiegs and Clark, 1963)	1-9	Computation, reasoning
Iowa Test of Basic Skills (Hieronymus & Lindquist, 1974)	1-8	Concepts, problem solving, computation
Metropolitan Achievement Tests 1. Survey Battery	1-12	Concepts, problem solving, computation
2. Criterion Referenced Inventory (Durost et al., 1978)		Concepts, problem solving, computation
Peabody Individual Achievement Test (Dunn & Markwardt, 1970)	K-12	Includes estimates of computation ability through multiple choice format, concepts, measurement, problem solving
Stanford Achievement Test (Madden, Gardner, Rudman, Karlsen & Merwin, 1973)	1-9	Concepts, computation and applications
SRA Achievement Series (Thorpe, Lefener & Mashind, 1969)	1-9	Concepts, reasoning, computation
Wide Range Achievement Test (Jastak & Jastak, 1965)	K-12	Computation
Woodcock-Johnson Psycho-Educational Test Battery (Woodcock & Johnson, 1977)	K	Concepts, mathematical reasoning, computation

TABLE 3

Diagnostic Arithmetic Tests

Name of Test	Grade Level	Skills Measured
Diagnostic Chart for Fundamental Processes in Arithmetic (Bushwell and John, 1925)	2 and above	Computational skill in the four fundamental processes
Diagnostic Tests and Self-Helps in Arithmetic (Brueckner, 1955)	2-8	Computational skill in four basic processes, common fractions, decimals and general arithmetic skills
Key Math Diagnostic Arithmetic Test (Connolly, Machtman and Pritchett, 1971)	K-6	Mastery of content, operations, and application
Stanford Diagnostic Arithmetic Test	2-8	Computation in fundamental processes, fractions, decimals

DISORDERS OF ARITHMETIC

Acalculia: Acquired Calculation Disturbances in Adults

Little is known about the nature or extent of children's arithmetic learning disabilities. Information from the study of acquired calculation disorders in adults has influenced some views of developmental arithmetic difficulties. Although caution is required in drawing analogies between these two fields of study, some insight into children's arithmetic disabilities may be gained from the more extensive exploration of adult disorders.

Acalculia is a descriptive term which designates an acquired disorder in calculation ability. The term was first applied by Henschen (1919) in an extensive survey of medical case reports of adult patients who displayed computational disorders subsequent to cerebral trauma. Henschen found that disorders in using numbers as words and in naming printed figures occurred in some cases without concomitant disorders in calculating ability per se. In other cases number-blindness without word-blindness was observed in direct association with acalculia. Acalculia was manifested in cases of occipital, frontal, parietal and temporal lobe lesions. Therefore, Henschen concluded that a) the integrity of several cortical areas is necessary for calculation and b) it is probable that there exists a distinct cortical network for arithmetic functioning.

Since this early compilation of clinical data, investigators have formulated taxonomies for categorizing diverse manifestations of acalculia, and have attempted to designate the site, or multiple sites, of cortical pathology responsible for the observed clinical signs and symptoms. Although

the literature of the field yields considerable variation and disagreement concerning both functional and anatomical groupings, there is basic agreement regarding three broad categorizations. These were first described by Hecaen (1962) and have been supported by Benson and Denckla (1969) and Benson and Weir (1972):

1. Aphasic acalculia - calculation errors emanating from an inability to handle numbers as words (attributed to dominant hemisphere lesions)
2. Visuo-spatial acalculia - calculation errors produced by an inability to align problems or to sustain place holding values (attributed to minor hemisphere lesions)
3. Anarithmetia - primary loss of calculation ability, inability in retrieval and/or manipulation of learned arithmetic values (associated with widespread pathology)

In his description of acalculic manifestations in more than forty adult patients, Cohn (1961) found evidence for two major distinct disturbances which were postulated to underlie the observed disordered calculating performance. The first involved disturbance in the processes of arithmetic order, while the second was associated with disturbance in memory processes. In those cases where the order defect was primary, manifestations included misalignment of vertical and horizontal number sequences and transposition of numerals. In those cases where a memory deficit was primary, the difficulties which predominated involved problems with number carrying, failure to follow the designated operational sign, and lack of memory for the multiplication tables.

Both Cohn's (1961) and Benson and Weir's (1972) investigations of adult acalculics focused primarily on computation of two and three digit multipli-

cation problems. Performance deficits on this task yielded the most striking manifestations of acalculia reported. Benson and Weir (1972) suggested that the striking defects evident in multi-digit multiplication performance of acalculic patients may be related to the particular demands of this task. They pointed out that two and three digit multiplication requires several different steps, to be performed almost simultaneously. Cohn (1961) noted that both ordering ability and memory are called upon in the performance of this task. Thus, each of these investigators has considered that a breakdown in performance of this complex task may reveal separate components of a calculating disturbance. While these separate components (e.g., number order, retrieval of multiplication facts) were not always in evidence when the arithmetic task was extremely simple, they were manifest when the task required them in combination. Study of arithmetic disorders in children suggests that these separate components may be manifested even on simple arithmetic tasks (Cohn, 1971).

Luria (1966) proposed that four varieties of arithmetic disturbances can be distinguished:

1. Primary acalculia - disintegration of the visual-spatial synthesis, loss of the categorical stability of number (associated with lesions in the parieto-occipital region)
2. Acoustic aphasic acalculia - inability to calculate aloud (associated with lesions of the left temporal region)
3. Motor aphasic acalculia - impairment in internal speech
4. Frontal lobe acalculia - impairment of performance of serial arithmetical operations, such as counting backwards

A major difference between Luria's categorizations and those of many other investigators appears to be in the prominence given visual-spatial factors.

Luria attributes primary acalculia to a breakdown in visual-spatial synthesis, while other investigators separate visual-spatial disturbance from other forms of acalculia. These other investigators consider primary acalculia, termed anarithmetia, to result from widespread cortical pathology, while visual-spatial acalculia is attributed to minor hemisphere lesions (Benson & Denckla, 1969; Benson & Weir, 1972; Hecaen, 1962).

Issues concerning the role of right and left hemisphere functions in calculation disturbances also have been discussed in the literature of acalculia. In his summary of findings relevant to this topic, Curry (1966) concluded that, while verbal functions are generally mediated in the left hemisphere and nonverbal functions in the right hemisphere, verbal ability facilitates nonverbal functioning to a considerable extent. Denckla (1975) has characterized the role of right and left hemispheres in arithmetic performance as involving an intricate "duet." Their respective roles are viewed, by Denckla (1975), as far more interdependent than might be expressed by the simplistic notion that one or the other side of the brain is most influential in calculation performance.

Gerstmann reported a cluster of unexpected symptoms in adults which he attributed to a circumscribed lesion in the parietal-occipital region of the left hemisphere (Grewel, 1952). The Gerstmann syndrome involves symptoms of bi-lateral finger agnosia, right-left confusion, agraphia and acalculia. Gerstmann emphasized the selective impairment in hand and finger differentiation (Grewel, 1952).

By the time Benton (1961) raised fundamental questions concerning the validity of the syndrome, the notion had already been extended to children's diffi-

culties of directionality and arithmetic learning in cases where no evidence existed for brain damage. The intriguing speculation that finger knowledge and finger differentiation might be intimately linked to competence in arithmetic gained currency among psychologists and anthropologists as well as among neurologists and neuropsychologists (Gardner, 1974). However, Benton's (1961) critical analysis led to the expression of reservations about the existence of the syndrome. Benton's major arguments were: 1) Gerstmann's case reports were altogether sketchy and vague, and 2) from Benton's own large-scale investigation, it was found that the four Gerstmann elements did not co-occur more frequently than other sets of symptoms.

Despite the conflicts which have surrounded this syndrome, the framework provided by Gerstmann has continued to influence investigations of acalculia. Even investigations of developmental arithmetic difficulties reflect this continuing influence (Kosc, 1974).

The diversity of calculating deficiencies, so often accompanied by or combined with language disorders, grapho-motor disturbances and other neurologically based dysfunctions, has made the study of adult acalculia both complex and problematic (Grewel, 1952). Acalculia is apparently not found in isolation from other manifestations of neurological disturbance (Cohn, 1961). Loss of calculating ability has been frequently observed in the course of routine neurological assessments of adult patients (Benson & Denckla, 1969), but is rarely the major presenting symptom in broad neurological diagnostic evaluation.

Aphasia represents a broader diagnostic category. Aphasia is a descriptive term which designates a disorder in the symbolic manipulation of language or a reduction of language facility (Schuell & Jenkins, 1961). Within the

study of aphasia considerable note has been taken of disturbances in calculation. Aphasic acalculia forms one of the generally accepted classifications of calculation disorders. This disturbance is characterized by difficulty with number language, by an inability to handle numbers as words.

Although aphasic acalculia represents a distinct classification, more general aphasic manifestations have been implicated in other types of acalculia. In their in-depth case report, Benson and Weir (1972) rule out both number language inadequacy and visuo-spatial disorientation as sources for the observed calculation disorder in a patient who also demonstrated dysnomia. Dysnomia, or unusual slowness and hesitation in naming upon confrontation, occurs to some degree in all aphasics (Gardner, 1974; Geschwind, 1967). Benson and Weir (1972) did not see a relationship between dysnomia and anarithmetia in the patient on whom they reported. However, several other investigators have suggested that dysnomia is implicated in diverse calculation disorders. Benson and Denckla (1969) considered that even "subtle or 'latent' aphasic symptoms may interfere with the direct performance of calculation tasks" (p. 102). On the basis of their clinical evaluations, Benson and Denckla proposed that acalculia should be considered within the context of aphasia, even when overt and obvious aphasic symptomatology is absent. Others have also viewed mild aphasic disturbances as implicated in a variety of calculation disorders (Lindquist, 1935; Poeck & Orgass, 1966; Singer & Low, 1933). Whether residual dysnomia directly affects diverse calculation disorders, or whether it merely accompanies them, remains unsettled.

The annals of adult neurology provide a considerable amount of information germane to understanding acquired disorders in calculation ability. Study of

loss of once intact calculating functions, attributable to demonstrable brain damage, has provided important information regarding how the adult human brain may function during the performance of arithmetic tasks. Despite the existing ambiguities and unknowns, the framework of adult acalculia has exerted influence on the ways in which some investigators have viewed children with arithmetic learning disabilities. Unfortunately, little empirical information has been gathered regarding the characteristics of children disabled in arithmetic.

Developmental dyscalculia

The literature on acquired acalculia has influenced the investigation of developmental arithmetic disabilities. Investigators have used the term developmental dyscalculia to designate disorders exhibited by some children in acquiring arithmetic competence. Dyscalculia denotes difficulty with calculating rather than calculating inability per se.

Cohn (1968) considered developmental dyscalculia as one manifestation of a neurological disorganization syndrome. He has proposed the following as manifestations of developmental dyscalculia:

1) Malformation or enlargement in the writing of number symbols
2) Strephosymbolia or twisted perception of images
3) Inability in the summing of single integers
4) Inability in the recognition of operator signs and in the use of lines to separate parts of a problem
5) Failure in the discrimination of specific order characteristics of multi-digit numbers

6) Inability in remembering and using multiplication tables

7) Inability in the "carrying" of numbers

8) Inappropriate sequencing of numbers in multiplication and division

He also contended that arithmetic is "one component of symbol operations" (p. 387). In his view, developmental dyscalculia is intimately connected to disorders of symbolic operations such as reading and writing.

Kosc (1974) defined developmental dyscalculia as:

> ...a structural disorder of mathematical abilities which has its origin in a genetic or congenital disorder of those parts of the brain that are the direct anatomico-physiological substrate of the maturation of the mathematical abilities adequate to age, without simultaneous disorder of general mental functions. (p. 47)

This definition places considerable emphasis on the neurological base of this disability and on the relative isolation of mathematical disturbance from other mental abilities. Kosc (1974) contended that his definition reflects the complexity of the disorder more accurately than does Cohn's (1971). Kosc offered an a priori taxonomy of developmental dyscalculia:

1) Verbal dyscalculia - disability in designating mathematical terms verbally and/or difficulty under conditions of oral presentation

2) Pragnostic dyscalculia - disability in the mathematical manipulation of real or pictured objects

3) Lexical dyscalculia - disability in reading mathematical symbols (digits, numbers, operational signs, etc.)

4) Graphical dyscalculia - disability in writing mathematical symbols

5) Ideognostical dyscalculia - disability in the understanding

of mathematical concepts and relationships

6) Operational dyscalculia - disability in the carrying out of operations, the interchange of one operation for another

The formulation of these categories resulted from Kosc's clinical work with acalculic adults and with children exhibiting arithmetic learning disabilities. Kosc's major empirical study of some 375 children failed to confirm the existence of the discrete categories which he proposed, and which are outlined above. It would seem that an adequate taxonomy has yet to be developed.

Kosc (1974) identified 6.4 percent of his total sample as being developmentally dyscalculic. Nearly half of those children who were identified demonstrated no neurological abnormalities. This finding is puzzling in view of Kosc's definition of developmental dyscalculia. Additionally, 17.6 percent of the total sample were considered to be mathematically inadequate. The study leaves unclear how those who were designated dyscalculic were distinguished from those who were mathematically inadequate.

The investigations of Cohn (1968, 1971) and Kosc (1974) represent the major empirical efforts to apply information from the field of adult neurology to the study of children's arithmetic difficulties. Formulations as to what constitutes developmental dyscalculia would appear to be incomplete and tentative at best. Arithmetic learning problems have received only minimal attention in the group of children classified as learning disabled, as well as in the population as a whole.

Factors in arithmetic learning disabilities

The current Federal definition of children with specific learning disabilities includes disorders of mathematical calculations:

> 'Specific learning disability' means a disorder in one or more of the basic psychological processes involved in understanding and using language, spoken or written, which may manifest itself in an imperfect ability to listen, think, speak, read, write, spell, or to do mathematical calculations. The term includes such conditions as perceptual handicaps, brain injury, minimal brain dysfunction, dyslexia, and developmental aphasia. The term does not include children who have learning problems which are primarily the result of visual, hearing or motor handicaps, of mental retardation, or of environmental, cultural or economic disadvantage (Federal Register, August 23, 1977).

Although children who manifest disorders in mathematical performance are specifically included under this definition, it is widely held that arithmetic difficulties rarely precipitate referrals for evaluation. School personnel often note that most referrals are made on the basis of children's difficulties in acquisition of reading skill. Goodstein and Kahn (1974) pointed out that even after learning disabled children have been identified, substantive arithmetic assessment or serious remedial programming rarely is provided.

Difficulties in acquiring arithmetic concepts and/or computational skills are probably as common and widespread as difficulties in other academic areas (Cohn, 1971; Kosc, 1974; Otto, McMenemy & Smith, 1973). Despite the apparently widespread nature of arithmetic learning problems, the data base available is "appallingly small" (Bryan & Bryan, 1978). Chalfant and Scheffelin's (1969) assertion that no systematic procedure exists for evaluating and diagnosing disorders in arithmetic still holds true ten

years later (Wallace & Larsen, 1978). There is also a dearth of programs, materials and methods specifically geared to the needs of children disabled in arithmetic (Hammill & Bartel, 1978; Padula, 1975). This state of affairs may reflect cultural values which have accorded preeminent status to reading achievement and literacy with a consequent lack of importance accorded mathematical development. Poor functioning in math may be quite socially acceptable, when not accompanied by similar poor performance on reading and writing tasks (Cohn, 1968; Denckla, 1975).

Johnson and Rising (1972) view mathematics as "concerned primarily with ideas, processes, and reasoning...mathematics is a way of thinking, a way of organizing a logical proof" (p. 3). Arithmetic, on the other hand, is most often seen as a special case of mathematics, as that branch of the science which deals with the understanding of real numbers and their computations (Brainerd, 1973; Farnham-Diggory, 1972). Chalfant and Scheffelin (1969) cautioned that disabilities in mathematics may be distinct from disabilities in arithmetic--that the cognitive processes required for mathematical reasoning may only partially intersect those processes required for competence in arithmetic. Drawing on her factor analysis of mathematical ability, Sister Canisia (1962) concluded that, although mathematical ability involves computational skills, computational facility need not be founded on mathematical ability.

Difficulties in the acquisition of arithmetic competence have been attributed to various influences. Mathematics educators consider that arithmetic difficulties result from the complexity of the subject matter, inadequate instructional methodology, and insufficient mastery of basic prerequisite skills (Cawley, 1978). Arithmetic achievement has been found to be corre-

lated with a number of "nonintellective" variables, including attitude, anxiety, interest, personality, sex, sociocultural background and teacher factors. In his survey of studies pertaining to the relationship between these variables and arithmetic achievement, Aiken (1970) acknowledged that these factors necessarily interact with "intellective" variables such as intelligence, verbal and spatial abilities.

Specialists in learning disabilities, while not altogether ignoring the influence of "nonintellective" variables, have tended to focus their concerns on cognitive factors. Chalfant and Scheffelin (1969) distinguished five factors which may significantly affect attainment of quantitative concepts: general intelligence, verbal ability, spatial ability, approach to problem-solving and neuro-physiological status.

Although the fundamental importance of general intelligence as a factor in mathematics performance has been amply demonstrated (Barakat, 1951), there is some evidence that it is more important in logical problem-solving than in simple arithmetic calculating (Schonell & Schonell, 1958). On the basis of several factor analyses, Wrigley (1958) concluded that high general intelligence is fundamental to success in mathematics. While normal intelligence is also considered basic to success in arithmetic (Denckla, 1975), its relative contribution to elementary arithmetic performance appears to vary considerably, depending upon the population investigated and the arithmetic criterion measures employed. For example, in a study comparing 46 learning disabled and 97 normal boys (Fleischner & Frank, 1979), I.Q. accounted for 18 percent of the variance in the normals' performance on one of the arithmetic criterion measures. Yet, IQ accounted for only 2.6 percent of the variance in the learning disabled boys' performance on this

same measure. An opposite pattern emerged when a different dependent measure of arithmetic was used: IQ accounted for only 3.1 percent of the variance in normal performance, while accounting for 37 percent of the variance in learning disabled performance. Within-group differences in the degree to which intelligence contributed to performance can be explained by the difference between the tasks: one measure stressed computation skills, while the other emphasized conceptual understanding. The between-group differences would appear to reflect different subject characteristics.

Goodstein and Kahn (1974) found arithmetic ability to be independent of level of intelligence within a group of 50 learning disabled children. Neither intelligence nor reading ability reliably predicted arithmetic performance in this group. The authors pointed out that the independence of these factors within a learning disabled population stands in marked contrast to findings of a close relationship among reading, arithmetic, and mental age within a population of educable mentally retarded children (Cawley & Goodman, 1968).

It seems evident that both subject differences and differences in the measures used to assess arithmetic performance will affect the degree to which intelligence contributes to arithmetic achievement. It is likely that the relative contribution of other psychological factors to arithmetic achievement might also vary as different populations are studied using different arithmetic measures. Therefore, it is important that normal samples be included in studies of exceptional groups, since extrapolations based on one population are not necessarily valid for another.

Verbal ability also has been found to be important to mathematical achievement. Wrigley (1958) contended that the high positive correlations evi-

denced between verbal measures and mathematical measures can be explained by their common correlation with general intelligence. Aiken (1972) took issue with this assertion, arguing that the observed high correlation between mathematical ability and general intelligence is "appreciably reduced" when linguistic abilities are partialed out. Thus, he considered language factors to be important to mathematics ability. Investigating factors related to quantitative ability in sixth graders, Muscio (1962) also supported the notion that arithmetic achievement depends upon verbal ability as well as on general intelligence.

Verbal ability is one of the five factors identified by Chalfant and Scheffelin (1969) as important in the development of quantitative thinking. They suggested that impairments in language functioning may interfere with the ability to organize and categorize information, which they view as fundamental to competence in arithmetic. Aiken (1972) emphasized the role of language factors in retention as well as in organization of arithmetic learning. While noting that mathematical performance is influenced by linguistic abilities, Aiken also proposed that mathematics itself is a specialized language. Others who have viewed arithmetic as a specialized language have noted the considerable degree to which it differs from "ordinary" language, thereby posing a veritable second-language dilemma for children (Nesher, 1972).

Cohn (1968) considered arithmetic ability to be part of the human system of symbolic operations, and, therefore, an aspect of language. On the basis of a longitudinal study of children with learning disabilities, he concluded that ability in arithmetic is neither better developed nor easier to achieve than other aspects of language (Cohn, 1971). Based on her

clinical observations of dyslexic children and adolescents, Ansara (1973) suggested that the same language difficulties which interfere with acquisition of reading skill also affect arithmetic performance. In particular, she noted difficulties with arithmetic vocabulary, difficulties in using language to guide computational steps and difficulties with the symbols used in arithmetic.

McLeod and Crump (1978) found that both verbal and visuo-spatial skills were significant factors in the arithmetic performance of 43 learning disabled children between the ages of seven and eleven. These children were selected from a larger sample of learning disabled children on the basis of their severe arithmetic difficulties. Two verbal measures, verbal meaning and auditory association, were found to be related somewhat more closely to the mathematical variable than was the visuo-spatial measure (Spatial Relations subtest of SRA Primary Mental Abilities Test). Based on these results, the authors concluded that investigation of the relationship between verbal abilities and arithmetic performance in children with learning disabilities is warranted.

Verbal ability, or language competence, seems to represent a significant factor in overall arithmetic achievement in normal individuals and in learning disabled children. The influence of particular language deficits on arithmetic performance has not yet been explored, although it has been suggested that the language deficiencies which may impede development in reading may also impede growth of skill in arithmetic. It would seem important to investigate the relationship between specific measures of language and particular aspects of arithmetic learning.

In addition to those factors just discussed, visual-perceptual and visual-

spatial abilities have also been cited as significant factors in mathematical ability (Barakat, 1951; Chalfant & Scheffelin, 1969). It has often been noted that children with learning disabilities exhibit perceptual deficits (Ayers, 1972; Frostig, 1972; Strauss & Lehtinen, 1947). Because of the high incidence of such deficits among learning disabled children and because of the importance accorded these children's reading difficulties, investigators have attempted to predict poor reading achievement from performance on visual-perceptual measures. In their review of the literature, Larsen and Hammill (1975) concluded that the weight of evidence does not support this relationship. At the same time, they noted some empirical evidence for the significant influence of visual perception in the arithmetic performance of learning disabled children.

Strauss and Lehtinen (1947) considered that arithmetic deficits result from lack of visual-spatial organization. They asserted that brain-injured children lack ability to discover spontaneously significant relationships of the number system. Although disturbance in visual-spatial organization was not viewed as the sole component of arithmetic learning difficulties, these authors considered it the most problematic to overcome. Kaliski (1962) also described children with arithmetic difficulty as displaying fundamental disturbance of spatial relationships. In addition, she suggested that motor disinhibition, impulsivity and language difficulties contribute significantly to their arithmetic deficiency.

More recently, Fleischner and Frank (1979) investigated the relationship between visual-perceptual ability and arithmetic achievement in 46 learning disabled and 97 normal boys in grades four through six. Their investigation revealed that, of the four variables which accounted for 26.5 percent of

the variance in learning disabled performance on the Wide Range Achievement Test (Jastak & Jastak, 1965), three were measures of visual-perception. Visual-perceptual measures also accounted for a significant portion of the variance in performance on the Peabody Individual Achievement Test, Arithmetic subtest (Dunn & Markwardt, 1970). The pattern of relationships among the predictor variables of IQ and visual perception and the two criterion measures of arithmetic performance differed in the normal sample. Even so, it was found that measures of visual-perceptual ability correlated significantly with arithmetic performance within this normal group. The authors concluded from these results that visual-perception is a probable good predictor of arithmetic achievement. The results of this study support the notion that perceptual functioning influences arithmetic achievement in learning disabled children. Further investigation in this area seems justified. In addition, it is also reasonable to pursue other possible correlates of arithmetic performance, as they may relate not only to overall arithmetic achievement, but also to specific sub-skills in arithmetic learning. Just as the cognitive processes required for mathematical reasoning may only partially intersect the processes required for overall arithmetic competence, so the processes necessary for a given arithmetic subskill may only partially intersect those processes required for general arithmetic achievement.

Basic number facts

Lack of computational skills has been cited in taxonomies of developmental arithmetic disability (Cohn, 1968; Johnson & Myklebust, 1964; Kosc, 1974). Mastery of basic number facts within whole number operations represents a computational subskill which has often been identified as problematic for learning disabled children. Practitioners frequently point out that, for many learning disabled children, basic number facts pose a serious

obstacle to mastery of arithmetic skills. Although these children can often compute the answers correctly, they continue to proceed slowly and laboriously long after their normal counterparts have developed considerable efficiency. Current investigations provide empirical support for learning disabled children's difficulty in developing proficiency with these basic number facts (Fleischner, Garnett and Shepherd, manuscript).

Basic addition and multiplication facts consist entirely of single-digit problems. Addition basic facts range from 0+0 through 9+9; multiplication basic facts range from 0x0 through 9x9. Basic subtraction and division facts are the reverses of these addition and subtraction problems. When commutatives (e.g., 3+4 = 4+3) are considered as separate facts, there are one hundred basic combinations for each of the four whole number operations.

In order to compute the answers to these problems, several prerequisite concepts have been viewed as fundamental: one-to-one correspondence, counting ability, conservation of quantity, a grasp of the meaning of the operations and knowledge of the symbols used to represent both quantity and operation. Once these basic concepts are acquired, children practice the computation of basic facts in order to attain computational proficiency. In pragmatic terms, proficiency with basic facts means that answers are computed accurately and swiftly enough to allow attention to be directed toward other aspects of complex arithmetic tasks.

Brownell (1935) identified a developmental sequence in the mastery of computation of basic number facts. From counting individual units of numbers, children progress through stages of grouping units. As counting decreases, children increasingly link one problem's solution to that of

another (e.g., 3+3 = 6, therefore 3+4 = 7). Finally, they habituate the response.

It has been proposed that learning disabled children may not develop the problem-linking strategies characteristic of the more advanced computational stage (Myers & Thornton, 1977; Thornton, 1978). Whether or not differences in the use of strategies are found between normal and learning disabled children, there remains a question regarding what psychological factors may contribute to basic fact inefficiency.

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